CURRENT RESEARCH INTEREST

- Application of Group Theory In
  - Probability Theory
  - Molecular Vibration
  - Fluid Mechanics & Flow Problem
  - Crystallography
  - Mathematical Physics
- Conjugacy and Order Classes of Groups
- Non-abelian Tensor Squares and Tensor Products of Groups
- Homological Functors of Groups
- Capability of Groups
- Lie Groups & Lie algebra
- Mathematical Modelling of Splicing Systems on DNA Molecules
- Solid Codes and Automata Diagrams in Splicing Systems
- Applied and Computational Complex Analysis
  - Numerical conformal mapping
  - Riemann-Hilbert problem

1. ON THE ABELIANNNESS OF SOME GROUPS

A group $G$ is abelian or commutative if its binary operation $*$ is commutative, that is, $ab = ba$ for all $a, b$ elements of $G$. However, not all groups are abelian, thus are called non-abelian group. It is interesting to raise this question: Can one measure in a certain sense how commutative a non-commutative group be?

In the past 20 years, and particularly during the last decade, there has been a growing interest in the use of probability in finite group that loosely called the “probability of $G$” by Dixon in 2004. In this research $2$-generator $p$-groups of nilpotency class 2 and $2$-Engel groups will be focused.

The idea of computing probability of $G$, $P(G)$, was introduced by Edros and Turan in 1968 who explored this concept mainly for symmetric groups. The same concept then used by MacHale in 1974 and Belcastro et. al. in 1994. It is obvious that the probability a pair of two elements chosen at random in a finite group $G$ is equal to one if and only if $G$ is abelian. They showed that, the probability is at most $5/8$ if and only if $G$ is a finite non-abelian group. Later, Rusin in 1979 chiefly looks the abelianness of finite group on nilpotency class 2.

There are two approaches on finding the probability that a pair of two elements commute, $P(G)$ that are by using Cayley Table or by using conjugacy classes. But, several author used the later approach.

$$P(G) = \frac{\text{Number of ordered pairs } (x, y) \in G \times G \text{ such that } xy = yx}{\text{Total number of ordered pairs } (x, y) \in G \times G}$$

$$= \frac{|\{(x, y) \in G \times G | xy = yx\}|}{|G|^2}.$$

$$= \frac{k(G)}{|G|}.$$
where $k(G)$ is the number of conjugacy class and $|G|$ is order of finite group $G$.

In 1979, Sherman introduced a theorem on the lower bound for the conjugacy class for any group $G$ which is

$$k(G) > \log_2 \log_2 |G|.$$  

Furthermore, Sherman have shown that if $G$ is a finite nilpotent group of nilpotency class $n$, then

$$k(G) \geq n|G|^{\frac{1}{n^n}} n + 1.$$  

Thus, by applying on the concept of probability,

$$P_{LB}(G) \geq \frac{n|G|^{\frac{1}{n^n}} n + 1}{|G|}$$

where $P_{LB}(G)$ is the lower bound probability of abelianness. Later, the $P_{LB}(G)$ will be generalized in this research for various groups.

Some related theorems and propositions will also be constructed in this research.

2. GROUP THEORETICAL APPROACH IN DETERMINING THE MOLECULAR VIBRATION OF SOME MOLECULES

The symmetry considerations are known to be an important tool for studying the behaviour of physical systems. The interrelationships of the symmetry properties of an object define a mathematical group. Mathematically speaking, the set of symmetry operations of a molecule or cluster forms a group and hence, can be treated by group theory.

Group theory, as applied to to the study of atoms, molecules or solids provides the connection between the symmetry of a system in real, three dimensional space, and the symmetries of the functions in Hilbert space which describe it mathematically. The application of the theory rests upon two pillars: (i) the group structure of the system’s set of symmetry operations. (ii) the nature of the vector spaces inhabited by the solutions to the system’s Schroedinger equation. These two pillars are blended into one in group representation theory. The significance of group theory for chemistry is that molecules can be categorized on the basis their symmetry properties, which allow the prediction of many molecular properties. The point group theory (and symmetry) provides the mathematical basis for interpretation of the spectra of molecules. The various vibrational modes of molecule can be categorized in terms of their behaviour with respect to the symmetry elements of the molecule.

A formulation of the structures and characteristics of molecules using the symmetry groups of 2- generator $p$-groups of nilpotency class two obtained provides a brand new research areas of research in Group Theory, resulting in applications in quantum chemistry or quantum mechanics. In this research, the symmetry groups of 2- generator $p$-groups of nilpotency class two obtained will be applied in chemistry particularly in molecular vibrations which utilizes much of the mathematical symmetry properties (i.e. angular dependencies) of its electronic and vibrational state functions.
3. APPLICATION OF 2-GENERATOR GROUPS OF NILPOTENCY CLASS TWO IN CRYSTALLOGRAPHY

Group Theory is a powerful formal method for analyzing abstract and physical systems in which symmetry is present. It is a beautiful area of mathematics that systematizes and formalizes mathematical study of symmetry. Some applications of Group Theory in Physics are, for example, crystallographic point groups. Group Theory plays a central role in the theory of molecules and crystalline solid where the groups are used to study the structures and the characteristics of crystals. What is more interesting is that Group Theory is playing an increasingly important part in providing algorithms which are amenable in quantum computing. It provides suitable functions for the calculation that can greatly reduce the effort involved.

In this study, our focus will be on the symmetric 2-generator 2-groups of nilpotency class two and its applications in crystallography.

4. HOMOLOGICAL FUNCTORS AND CAPABILITY OF VARIOUS GROUPS

The nonabelian tensor square is a special case of nonabelian tensor product of groups which has its roots in algebraic K-theory as well as topology. Earlier work by Miller in 1952 and Dennis 1976 is in context with Schur multipliers and algebraic K-theory, respectively are referred.

Recent work of Beurle and Kappe in 1999 focuses the homological functors only on infinite metacyclic groups. This research will cover the applications on a wider basis which includes all two-generator groups of nilpotency class two and more. The computation of various functors of various groups of nilpotency class two includes the exterior square, the symmetric product and the second homology of these groups. The research will also be carried out in determining which of these groups are capable.

5. IRREDUCIBLE REPRESENTATIONS, CONJUGACY AND ORDER CLASSES OF SOME 2-GENERATOR p-GROUPS OF NILPOTENCY CLASS 2

Most applications of group to physical problems are, more particularly, applications of representation theory. Representation theory has been used extensively in many applied fields such as spectroscopy, crystallography, quantum mechanics and molecular orbital theory. The main part of studying representation theory is to look at the irreducible representations of the group and their order classes. Order classes is a set of elements which have the same order. In this research, the number of irreducible representations or conjugacy classes of 2-generator p-groups of nilpotency class 2 will be generalized. Furthermore, given the order classes, specific groups in the classification will be determined.
6.  **ON THE NONABELIAN TENSOR SQUARES OF CERTAIN BIEBERBACH GROUPS WITH CYCLIC POINT GROUPS**

The Bieberbach groups are torsion free crystallographic groups. A crystallographic group $G$ is a group extension of a group $P$ by a free abelian group $V$ of finite rank $d$. Hence there is a short exact sequence of the form

$$0 \rightarrow V \rightarrow G \rightarrow P \rightarrow 1$$

The group $V$ is called the lattice subgroup and $P$ is called the point group or holonomy group of $G$. The rank $d$ of $V$ is called the dimension of $G$.

The goal of this research is to compute the nonabelian tensor squares of certain Bieberbach groups of dimension $d$ with cyclic point groups. These groups are metabelian polycyclic groups. A theory for computing the nonabelian tensor squares of polycyclic groups has been developed by Blyth and Morse. This method will be used to compute the nonabelian tensor squares of two families of Bieberbach groups whose nonabelian tensor squares are abelian as well as some other families of Bieberbach groups.

Using CARAT, there are 28 non-isomorphic Bieberbach groups when $P$ is cyclic group of order 2 (up to dimension 6) and represented as matrix group. Using GAP, these matrix groups are converted into finitely presented groups and polycyclic groups. All polycyclic representations and the relations which are consistent to the pcp representation of these groups are obtained. These groups are investigated based on their nonabelian tensor square. Some of properties of nonabelian tensor square of these groups have been obtained and successfully generalized to $n$ dimension.

From the computation, the nonabelian tensor square of these groups up to $n$ dimension can be divided into two types: abelian and nonabelian. For the abelian nonabelian tensor square, we found that there are exactly two families Bieberbach groups with point group $C_2$ of dimension $n > 2$ and their group presentation have been obtained.

A general formulas to calculate the nonabelian tensor square of Bieberbach group with point group $C_2$ (abelian cases) for $n$ dimension for groups obtained in 2 will be constructed. Theorems and lemmas related to the nonabelian tensor squares of these group will be developed and some of them have been proved.

7.  **COMPUTING THE NON ABELIAN TENSOR SQUARES OF CERTAIN BIEBERBACH GROUPS BY THE DIHEDRAL POINT GROUP OF ORDER 8**

A Bieberbach group is any torsion free extension of free abelian infinite cyclic group of rank $n$ by some finite point group.

The goal of this research is to compute all nonabelian tensor squares of centerless Bieberbach group with point group isomorphic to the dihedral group of order 8.

Firstly, the properties of the Bieberbach group will be determined. Then, the nonabelian tensor squares of the group for small rank $n$ will be computed and studied in order to get the pattern. Finally general theories of the nonabelian tensor square of the Bieberbach group by the dihedral group of order 8 will be determined. These will be done with the help of crystallographic algorithms and tables package (CARAT) and using Group, Algorithm and Programming (GAP) software.
8. **MATHEMATICAL MODELLING OF SPLICING SYSTEMS ON DNA MOLECULES**

Every living organism has DNA that makes the organism unique. Since a DNA strand can be viewed as a string over a four letter alphabet (a, c, g, and t) which is the four deoxyribonucleotides, thus the modelling can be done within the framework of formal language theory, which is a branch of applied discrete mathematics and theoretical computer science.

Splicing system was defined to model the recombinant action of restriction enzyme and a ligase on DNA molecules. They were originally developed as a mathematical or dry model of the generative capacity of DNA molecules in the presence of appropriate enzymes. This theory relates formal language theory to the study of informational macromolecules. In this research the sequence data for a DNA molecule serves as the formal representation of the molecule. The language which results from a splicing system is called a splicing language. This language contains the initial strings of DNA molecules and is closed under the application of splicing rules.

A splicing language can consist of two types of languages which are adult language and limit language. Adult language consists of the strings that are produced by the system but do not participate in further operations. Limit language consists of those strings that will be present after a splicing system has reached equilibrium, without considering whether or not they will participate in further operations. A wet-lab procedure which will illustrate a simple possible case in which the adult language and limit language are distinct is initiated. This experiment will produce a laboratory verification of Head's "dry model" of the actual wet-lab procedure.

The most specific benefit of this research will be the application of mathematical analysis of which DNA biomolecules can potentially arise in a test tube from the action of specific sets of enzymes acting on the specific sets of DNA molecules.

9. **SOLID CODES IN SPLICING SYSTEMS**

Splicing system was first introduced by Tom Head in 1987 as the mathematical model of systems of restriction enzymes acting on initial DNA molecules. In this research, some new concepts of solid codes are presented. For an $S_nH$ system, where $k$ is the length of the longest word in a rule $R$, it may be reduced to a simple splicing system if $R$ is a solid code. Examples are given to show that splicing systems having solid rules can be reduced to simple splicing systems by replacement of the occurrences of words in the rule $R$ by letters of an alphabet of new letters.

Besides, the concept of firm and maximal firm subwords are also introduced. Some examples are given to illustrate the maximal firm subwords of a word in a simple splicing system. Simple splicing systems can also be recognized by $SH$-automata diagrams due to the regularity of splicing languages. Therefore, $SH$-automaton defines exactly one language which is the language generated by the simple splicing system. Taking the $SH$-automata concept, the maximal firm subwords of the initial words of an $SH$ systems serve as the labels for the associated $SH$-automaton. Some examples which will show the maximal firm subwords of the words in the initial set $I$, the regular expression for the language generated by the given splicing system and the simplest non-deterministic automaton that recognizes the corresponding splicing system are also given.

10. **GENERALIZATION OF RULES IN SPLICING LANGUAGES**

In this research, different types of splicing system will be studied together with their properties. These splicing systems will then be connected using their rules. Besides, generalization of splicing languages will be made based on their rules, and proofs regarding these generalizations will be given.
11. NUMERICAL CONFORMAL MAPPING

An important and familiar tool of science and engineering since the development of complex analysis is conformal mapping. Conformal mapping uses functions of complex variables to transform complicated boundaries to simpler, more manageable configurations. In various applied problems, by means of conformal maps, problems for certain "physical regions" are transplanted into problems on some standardized "model regions" where they can be solved easily. By transplanting back we obtain the solutions of the original problems in the physical regions. This process is used, for example, for solving problems about fluid flow, electrostatics, heat, mechanics, and aerodynamics. Several theoretical and numerical results have been obtained by this research group for conformal mapping of simply and multiply connected regions via the integral equation methods involving the Kerzman-Stein kernel, the Neumann kernel, and the generalized Neumann kernel.

![Figure 1](image1.png)

Figure 1: Interior region of an ellipse using polar coordinates.

![Figure 2](image2.png)

Figure 2: Riemann map of the interior region of an ellipse using integral equation method.

12. RIEMANN-HILBERT PROBLEM

This research group also considers an important complex boundary value problem that has an application in hydrodynamics known as the Riemann-Hilbert problem. The classical boundary value problems such as the Dirichlet problem and the Neumann problem can be reformulated as a Riemann-Hilbert problem. This research group has obtained new Fredholm integral equations associated to the interior and exterior Riemann-Hilbert problems in simply and multiply connected regions. The kernel of these integral equations is the generalized Neumann kernel.
Further work has been done by this research group to discuss the solvability (i.e., existence and uniqueness) of the integral equation and its extension to non-smooth regions and multiply connected regions. As an application, based on the integral equation for exterior Riemann-Hilbert problem, we have derived a new boundary integral equation for external potential flows. The developed method has been used to compute numerically the flows past a single and multiple airfoils.

Figure 3: Flows past a single airfoil.